

A Probabilistic Approach to the Simulation of Non-Linear Stress-Strain Relationships for Oriented Strandboard

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Abstract

This paper presents preliminary results of a study that is seeking to develop a stochastic finite element model to predict the global structural response when loaded in four-point bending of different OSB panel designs based on the physical and mechanical properties. Experimental testing has been conducted to evaluate the physical and mechanical properties on a selection of OSB panels in a variety of thicknesses from three different producers. Statistical analyses have been conducted to establish relationships to describe the stress-strain behaviour, to determine the underlying probability distribution models to describe each property. Regression analyses have been conducted to identify relationships between different properties.

Keywords Oriented strandboard, OSB, wood-based panels, optimisation, mechanical properties, stress-strain relation

Introduction

Oriented strandboard (OSB) is a wood based composite material made from 3 layers of elongated wood strands coated in a thermosetting resin binder that are hot pressed to form large structural panels. OSB can be made from low-grade, under-utilised forest resources such as small diameter logs from tops and thinnings and fast-growing, low-density wood species, offering significant environmental and forest resource management benefits over traditional wood products. The natural variability of the raw materials means that both the physical and mechanical properties of OSB are inherently variable and difficult to predict. OSB product development and quality control is heavily reliant on expensive, time-consuming empirical methods with production parameters being manipulated based on empirical data.

This paper presents the preliminary output of a study seeking to develop a method of predicting the mechanical response of OSB. The research presented within follows-on from previous work by the authors on a similar investigation conducted on OSB subject to in-plane tension (McTigue and Harte, 2010). A variety of thicknesses of OSB/3 panels, produced by three different manufacturers, were tested using standard four-point bending arrangements. The results have been used to establish stress-strain relationships to describe the mechanical behaviour up to failure, to determine appropriate probability distribution models to describe the system variables and to identify relationships between variables within the system. Past studies have largely concentrated on predicting the mechanical behaviour of wood-based composites based on the mechanical behaviour of the raw materials with model verification being achieved through experimental testing of small scale, laboratory produced panels. This study is focusing on predicting the mechanical properties of existing, commercially available panels based on physical properties.

Literature Review

A review of the use of probability based methods in the forest products industry conducted by Taylor, et al. (1995) demonstrated the effectiveness of this approach to accurately predict the mechanical behaviour of structural wood systems. The Monte-Carlo method has proven to be a particularly useful tool when it comes to modelling wood-based composite materials. The effectiveness of the Monte-Carlo method is however dependent on knowledge of the underlying probability distribution of each variable in the system and identification of relationships between variables within the system. Hunt and Suddarth (1974) developed a 2-D linear elastic finite element model to predict the elastic stiffness of a single-layer, random flakeboard when loaded in tension and panel-shear. The board was modelled as a regular grid of beam elements (representing the binder) infilled with plate elements (representing the flakes). The random distribution of the wood-strands in the board was simulated using the Monte-Carlo method to independently assign a random flake orientation to each plate based on a uniform probability distribution model. The average predicted tension modulus of elasticity (MOE) differed from the

experimental value by 2% to 3% while the average predicted shear modulus differed from the experimental value by 10% to 12%.

Wang and Lam (1998) developed a 3-D non-linear stochastic finite element model for wood-based composites that incorporated several probability based techniques to predict the probabilistic distribution of the tension strength and MOE of multi-layered parallel aligned wood strand composites. The model input was generated through testing of individual wood strands with standardised dimensions to evaluate the tension strength and MOE, to determine the underlying probability distributions of each variable and to identify relationships between variables. Assemblies of strands were tested at a longer gauge length and the results were used for comparison with model predictions. The Monte-Carlo method was used to randomly assign material properties to individual strands based on the underlying probability distributions and the effect of the increased volume was accounted for using the Weibull weakest link theory. The relationship between tension strength and MOE was preserved in the Monte-Carlo simulation using the standard bivariate normal distribution (Lam, et al., 1994; Wang, et al., 1995). Excellent agreement was achieved between the simulated and experimental probability distributions for tension strength and MOE of the multi-ply laminates. Clouston and Lam (2001) developed the model further to enable it to predict the mechanical response of angle-ply wood strand laminates subjected to multiaxial stress conditions. Excellent agreement was observed between the predicted and experimental probability distributions for ultimate strength and MOE in tension, compression and bending. Subsequent studies by the same authors (Clouston, 2001; Clouston and Lam, 2002) elaborated the model into a 3-D non-linear stochastic finite element model capable of predicting the probabilistic distribution of strength, stiffness and failure load of angle-ply laminates subjected to tension, compression and bending. Further development expanded the model's capabilities to predict the probabilistic distributions of strength and MOE of large-section parallel strand lumber members loaded in tension, compression and bending (Clouston, 2007).

Testing

Materials

The materials tested were commercially available OSB/3 panels manufactured in accordance with BS EN 300:2006 (BSI, 2006). Sample sizes for each panel manufacturer, thickness and material property direction are given in Table (1). Panels were produced by Manufacturer A using Sitka spruce and Scots pine wood strands bound with MDI resin stacked in a 0-90-0 lay-up pressed in a daylight press. Panels were produced by Manufacturer B using Scots pine and Lodgepole pine wood strands bound with MUPF resin in the surface layers and PMDI resin in the core stacked in a 0-90-0 lay-up pressed in a daylight press. Panels were produced by Manufacturer C using pine wood strands bound with MUPF resin in the surface layers and pMDI resin in the core layer stacked in a 0-90-0 lay-up pressed in a continuous press.

Specimen Preparation

A total of 32 cutting plans were prepared for each panel thickness in accordance with the guidelines in BS EN 789:2004 (BSI, 2004) of which 15 were selected at random. Test pieces cut with their longer dimension aligned parallel to the longer dimension of the panel are designated

longitudinal (LONG) while test pieces cut at 90° to the longer dimension of the panel are designated lateral (LAT). Test pieces were conditioned at 20°C and 65% relative humidity prior to testing. The dimensions of the test pieces for four-point bending varied with panel thickness as per the guidelines in BS EN 789:2004 (BSI, 2004).

Table 1 – Sample Sizes

	Manufacturer A		Manufacturer B		Manufacturer C	
Thickness	Longitudinal	Lateral	Longitudinal	Lateral	Longitudinal	Lateral
11mm	15	15	-	-	-	-
15mm	15	15	15	15	15	15
18mm	15	15	-	-	-	-

Figure 1 - Sample Cutting Plan

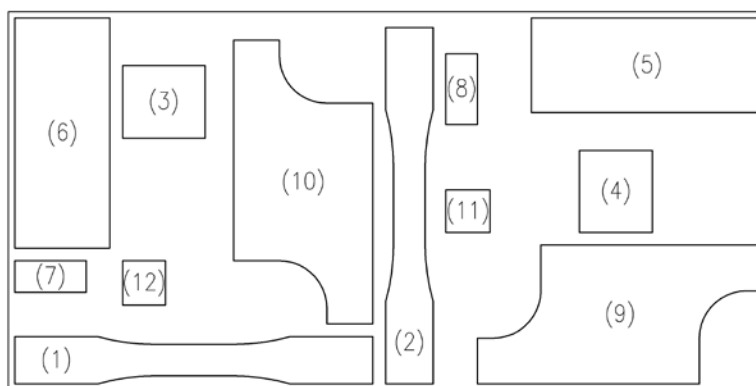


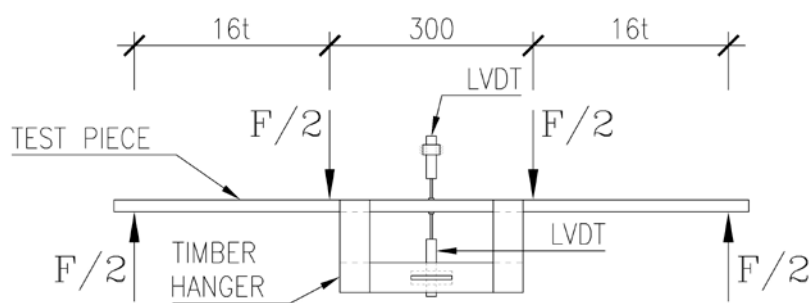
Table 2 – Test Piece Details

Material Property	Test Piece Number	Direction	Bending Test Piece Dimensions		
Tension	(1)	Longitudinal			
	(2)	Lateral			
Compression	(3)	Longitudinal			
	(4)	Lateral			
Bending	(5)	Longitudinal			
	(6)	Lateral			
Planar Shear	(7)	Longitudinal			
	(8)	Lateral			
Panel Shear	(9)	Longitudinal	Thickness	B (mm)	L (mm)
	(10)	Lateral	11mm	300	652
Bearing	(11)	Longitudinal	15mm	300	780
	(12)	Lateral	18mm	300	876

Test Setup and Procedure

Testing was conducted using an Instron 4466 universal screw-press testing machine with a 10kN load cell with an accuracy of $\pm 1\%$. The test setup (see Fig. 2) was in accordance with the specifications in BS EN 789:2004 (BSI, 2004).

Figure 2 – Schematic Four-Point Bending Test Setup



The local deflection in the region between two loading points was recorded using two full-bridge linear voltage differential transducers (LVDT's) with a 25mm ram and an accuracy of $\pm 1\%$ mounted to the underside of the test piece using a hanger. An additional set of analogue LVDT's with a 100mm ram and an accuracy of $\pm 1\%$ mounted to the frame of the test machine were used to record the global deflection over the full span. Load was applied using a constant rate of strain such that that average time of to failure was 300 ± 120 s. The test piece was initially loaded to 50% of its expected failure load, followed by unloading and removal of the hanger system. The load was re-applied to failure with the two analogue LVDT's still in position to monitor the global deflection.

Results

Mechanical Properties

The true bending MOE was calculated from the local deflection results using Equation (1) below while the bending strength was calculated using Equation (2) (BSI, 2004). In addition, the global bending MOE was calculated from the global deflection results using Equation (3) below while an estimate the out-of-plane shear modulus was calculated using Equation (4) below. Summary statistics (including mean, 5th percentile and coefficient of variation (COV) (BSI, 1995)) for bending properties are presented in Tables (3) and (4) for each panel manufacturer, thickness and material property direction. The results show that significant orthotropy exists in mechanical properties with significantly enhanced performance when the panel longitudinal direction is aligned parallel to the span direction.

$$E_b = \frac{(F_2 - F_1)l_1 l_2}{16(u_2 - u_1)I} \quad \text{Equation (1)}$$

$$f_b = \frac{F_{max} l_2}{2W} \quad \text{Equation (2)}$$

$$E_{b,G} = \frac{(F_2 - F_1)l_1 l_2}{4bt^3(u_{2,G} - u_{1,G})} \quad \text{Equation (3)}$$

$$G_v = \frac{6E_b I F_{max}^2 l_2}{A(48E_b I \delta_{max,MOR} - F_1 l_2 (3L^2 - 4l_2^2))} \quad \text{Equation (4)}$$

Where: E_b = true MOE; f_b = strength; F_2 = load at $0.4F_{max}$; F_1 = load at $0.1F_{max}$; u_2 = displacement corresponding to F_2 ; u_1 = displacement corresponding to F_1 ; F_{max} = failure load; l_1 = hanger gauge length; l_2 = distance from loading points to supports; I = second moment of area; W = elastic section modulus; $E_{b,G}$ = global MOE; $u_{2,G}$ = global displacement corresponding to F_2 ; $u_{1,G}$ = global displacement corresponding to F_1 ; t = thickness ; b = breath; L = total span; $\delta_{max,MOR}$ = deflection at failure; G_v = out-of-plane shear modulus.

Table 3. Bending Test Results (Strength and True MOE)

Panel	Longitudinal						Lateral					
	Strength			True MOE			Strength			True MOE		
	Mean (N/mm ²)	5 th Percentile (N/mm ²)	CoV (%)	Mean (N/mm ²)	5 th Percentile (N/mm ²)	CoV (%)	Mean (N/mm ²)	5 th Percentile (N/mm ²)	CoV (%)	Mean (N/mm ²)	5 th Percentile (N/mm ²)	CoV (%)
A-11mm	30.79	24.70	14.96	10192	7064	18.74	21.25	15.58	9.04	4749	4259	8.81
A-15mm	29.44	23.55	12.52	10050	8330	12.61	18.65	16.43	12.84	4357	3566	10.16
A-18mm	25.88	21.93	14.12	8725	7138	12.22	15.26	12.89	8.91	3480	2784	16.65
B-15mm	28.38	25.01	8.33	8635	7617	9.11	18.60	14.92	15.40	3636	3066	11.23
C-15mm	24.59	18.66	13.47	9938	7822	17.03	15.54	12.22	14.60	3729	3135	12.51

Table 4. Bending Test Results (Global MOE and Estimated Out-of-Plane Shear Modulus)

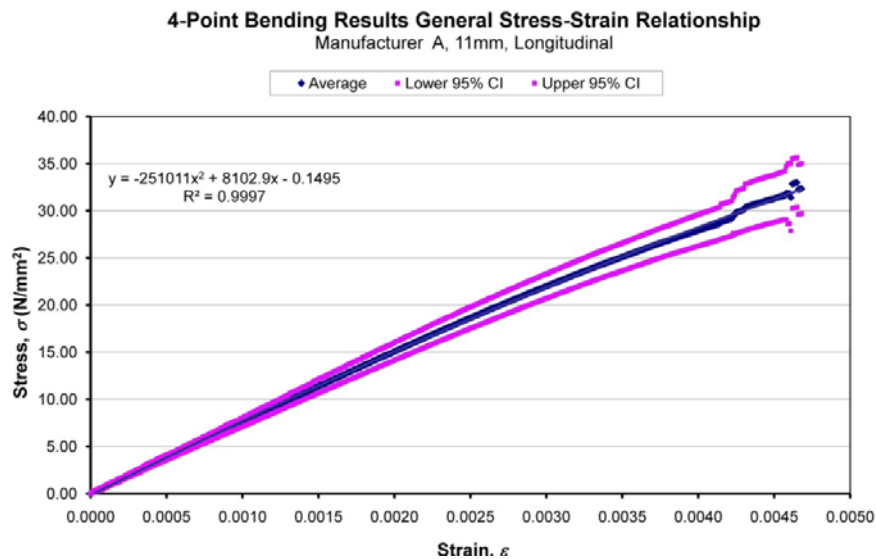
Panel	Longitudinal						Lateral					
	Global MOE			Out-of-Plane Shear Modulus			Global MOE			Out-of-Plane Shear Modulus		
	Mean (N/mm ²)	5 th Percentile (N/mm ²)	CoV (%)	Mean (N/mm ²)	5 th Percentile (N/mm ²)	CoV (%)	Mean (N/mm ²)	5 th Percentile (N/mm ²)	CoV (%)	Mean (N/mm ²)	5 th Percentile (N/mm ²)	CoV (%)
A-11mm	7612	6111	14.4	3165	1660	50.42	4088	3763	4.91	1120	659	44.53
A-15mm	6912	6050	7.52	3561	2620	22.4	3356	3043	6.38	1163	886	21.82
A-18mm	6468	5592	12.25	4419	2658	29.68	2911	2516	7.77	1412	966	34.55
B-15mm	6500	6007	6.02	3944	3077	15.86	3197	2713	8.81	1483	825	38.90
C-15mm	7057	5745	12.31	3178	1128	35.50	3116	2906	11.77	843	552	32.88

Regression Analysis

Linear equations in the form $\sigma = a\varepsilon + b$ as well as quadratic stress-strain equations in the form $\sigma = a\varepsilon^2 + b\varepsilon + c$ were fitted to the global stress-strain data for each specimen tested. The results indicated that while the linear equation describes the relationship between stress and strain at lower strains, the quadratic equation more accurately describes the relationship between stress

and strain over the full range. Average stress-strain curves were generated for each sample based on the quadratic model. The average stress-strain curves were generated for each sample from the global stress-strain data by averaging the stress along lines of constant strain at the extreme fibre on the tension face (Clouston and Lam, 2001). Figure (3) shows a typical average stress-strain curve and the associated 95% confidence interval (Hayter, 2002; Levine, et al., 2001) for 11mm thick panels produced by Manufacturer A with the longitudinal direction aligned parallel to the direction of the span.

Figure 3 – Average Stress v's Global Strain Curve



Probability Distribution Model Fitting

A computer program was written using the Microsoft Visual Basic for Applications (VBA) programming language to automatically determine the most suitable model to describe the experimental results. The output included probability plots for visual inspection of the goodness-of-fit between the empirical distribution function (EDF) and cumulative distribution function (CDF) for each probability distribution being examined. The Anderson-Darling test was used as a scientific basis for choosing the most suitable probability distribution model. The Anderson-Darling test is the most robust of the goodness-of-fit tests for determining the underlying probability distribution for both small and large samples (D'Agostino and Stephens, 1986; Stephens, 1974). The data is ranked in ascending order and the Anderson-Darling statistic is calculated using Equation (5) and modified according to sample size using Equation (6). The probability (P -value) that accepting the null hypothesis can be calculated based on the value of A^2 . The method of calculating P depends on the probability distribution being examined and the value of A^2 . Detailed sets of formulae for calculating the P -value can be found in (D'Agostino and Stephens, 1986). A high P -value indicates there is strong evidence to suggest that the sample comes from a population that follows the probability distribution being examined. Figure (4) shows a typical cumulative probability plot for the bending strength results for the 15mm thick panels produced by Manufacturer A with the longitudinal direction aligned parallel to the direction of the span.

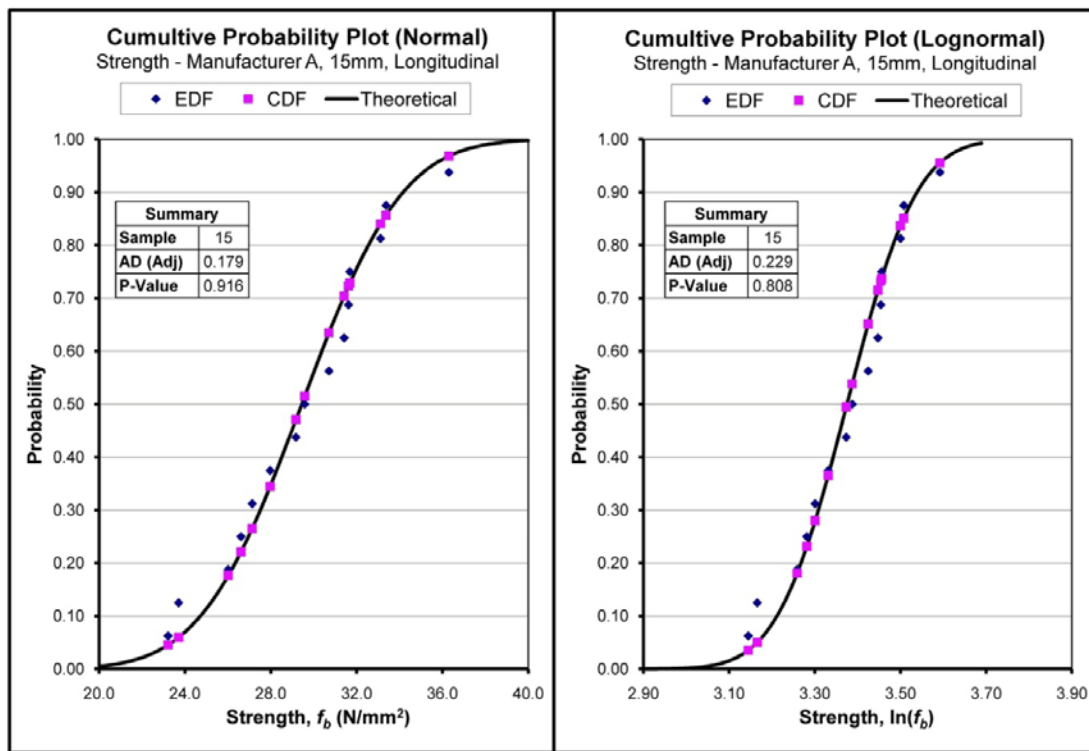
$$A^2 = -N - S \quad \text{Equation (5)}$$

$$A^2_{Adj} = A^2 \left(1 + \frac{0.75}{N} + \frac{2.25}{N^2} \right) \quad \text{Equation (6)}$$

Where: N = sample size; S is given by Equation (7) below; $F(Y_i)$ = CDF of probability distribution evaluated at observation Y_i ; $F(Y_{N+1-i})$ = CDF of probability distribution evaluated at observation Y_{N+1-i} .

$$S = \sum_{i=1}^N \frac{2i-1}{N} [\ln F(Y_i) + \ln(1 - F(Y_{N+1-i}))] \quad \text{Equation (7)}$$

Figure 4 – Typical Cumulative Probability Plots



The plot shows the EDF for the sample results and the CDFs for normal and lognormal probability distributions. A summary table containing the sample size, the A^2 value and the corresponding P -value is included on each chart. Visual inspection indicates that both probability distribution models describe the data quite well. Referring to Figure (4), the P -value for the lognormal distribution is 0.916 whereas the P -Value for the normal distribution is 0.808, indicating that the normal probability distribution is a better fit. Tables (5) and (6) contain the A^2 values and the corresponding P -values for the bending properties for each panel and material

property direction. The results demonstrate that the mechanical properties of the panels tested can generally be described by either the normal or lognormal probability distributions. No conclusion could be drawn in one circumstance most likely because of an insufficient number of test replications to capture the underlying probability distribution.

Table 5 – Anderson-Darling Test Results (Strength and True MOE)

	Results Set	Strength					True MOE				
		Normal		Lognormal		Conclusion	Normal		Lognormal		Conclusion
		A^2_{adj}	P -Value	A^2_{adj}	P -Value		A^2_{adj}	P -Value	A^2_{adj}	P -Value	
Longitudinal	A-11mm	0.4266	0.3138	0.4144	0.3354	Lognormal	0.3117	0.5515	0.4497	0.2765	Normal
	A-15mm	0.1797	0.9167	0.2297	0.8080	Normal	0.3780	0.4078	0.3674	0.4314	Lognormal
	A-18mm	0.4189	0.3273	0.3704	0.4246	Lognormal	0.1528	0.9595	0.1211	0.9885	Lognormal
	B-15mm	0.2561	0.7245	0.2899	0.6128	Normal	0.3638	0.7245	0.2899	0.6128	Normal
	C-15mm	0.5177	0.1887	0.6610	0.0787	Normal	0.3877	0.3872	0.3203	0.5321	Lognormal
Lateral	A-11mm	0.4486	0.2782	0.4877	0.2237	Normal	0.5549	0.1523	0.5504	0.1564	Lognormal
	A-15mm	0.6397	0.0889	0.5206	0.1856	Lognormal	0.2824	0.6367	0.3615	0.4451	Normal
	A-18mm	0.2264	0.8175	0.2988	0.5860	Normal	0.3238	0.5252	0.2426	0.7685	Lognormal
	B-15mm	0.3892	0.3841	0.6152	0.1023	Normal	0.2103	0.8604	0.2145	0.8498	Normal
	C-15mm	0.3150	0.5437	0.4462	0.2819	Normal	0.2699	0.6781	0.2582	0.7175	Lognormal

Table 6 – Anderson-Darling Test Results (Global MOE and Estimated Shear Modulus)

	Results Set	Global MOE					Estimated Out-of-Plane Shear Modulus				
		Normal		Lognormal		Conclusion	Normal		Lognormal		Conclusion
		A^2_{adj}	P -Value	A^2_{adj}	P -Value		A^2_{adj}	P -Value	A^2_{adj}	P -Value	
Longitudinal	A-11mm	0.2843	0.6305	0.2664	0.6899	Lognormal	1.542	0.0004	0.9392	0.0159	Inconclusive
	A-15mm	0.2635	0.6997	0.2998	0.5831	Normal	0.2332	0.7976	0.1359	0.9781	Normal
	A-18mm	0.4172	0.3303	0.3645	0.4381	Lognormal	0.1629	0.9448	0.2848	0.6298	Normal
	B-15mm	0.4767	0.2380	0.4249	0.3168	Lognormal	0.5631	0.1452	0.5712	0.1385	Normal
	C-15mm	0.7576	0.0452	0.8585	0.0253	Inconclusive	0.3957	0.3710	0.3166	0.5401	Lognormal
Lateral	A-11mm	0.7379	0.0506	0.7771	0.0404	Normal	1.1204	0.0056	0.4867	0.2250	Lognormal
	A-15mm	0.3931	0.3762	0.3542	0.4625	Lognormal	0.8369	0.0287	0.6533	0.0823	Lognormal
	A-18mm	0.3972	0.3680	0.4586	0.2632	Normal	1.3396	0.0016	0.7263	0.0541	Lognormal
	B-15mm	0.2446	0.7622	0.3008	0.5803	Normal	0.3936	0.3752	0.2463	0.7567	Lognormal
	C-15mm	0.1877	0.9031	0.1968	0.8888	Normal	0.6423	0.0876	0.3236	0.5256	Lognormal

Conclusions

The results of this test program and the statistical analysis of the results indicate that the global bending stress-strain behaviour of OSB can be described by a quadratic expression up to the point of failure. Average stress-strain relationships have been established for each panel type, thickness and material property direction along with the associated 95% confidence intervals. Visual comparison of probability plots indicates that the bending strength and tension MOE can

be reasonably well represented by either a normal or lognormal probability distribution while the Anderson-Darling test has been shown to be effective at selecting the probability distribution model that best describes the data.

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