PARABOLIC BEHAVIOR OF THE YOUNG'S MODULI RATIO OF WOOD OBTAINED BY VIBRATION TESTS WITH TIME DURING DRYING

*Yoshitaka Kubojima**

Researcher
E-mail: kubojima@ffpri.affrc.go.jp

Youki Suzuki

Researcher
E-mail: youki@ffpri.affrc.go.jp

Hideo Kato

Researcher
E-mail: KATO_Hideo@ffpri.affrc.go.jp

Isao Kobayashi

Researcher
Forestry and Forest Products Research Institute
Tsukuba, Japan
E-mail: kobayas@ffpri.affrc.go.jp

(Received October 2016)

Abstract. A method to estimate the water distribution within wood lumber was developed using bending and longitudinal vibration tests. Sitka spruce and Japanese cedar green wood were left inside a controlled environment of 20°C and 65% relative humidity. The bending and longitudinal vibration tests were performed and the Young's modulus was obtained by each testing method. And then, the temporal changes in the ratio of Young's modulus based on the bending vibration to that based on the longitudinal vibration were investigated during drying. The ratio of Young's modulus increased and then decreased with time during drying due to the difference between the Young's modulus of the outer portion of wood and that of the inner portion. The temporal change in the ratio of Young's modulus was similar to the estimate using a simple cross-sectional model developed in this study. The bending Young's modulus calculated using Euler-Bernoulli's elementary theory could be used instead of that using the Goens-Hearmon regression method based on the Timoshenko theory of bending.

Keywords: Bending vibration, longitudinal vibration, temporal change, TGH method, water distribution, Young's modulus.

INTRODUCTION

Electric and near-infrared spectroscopic methods are nondestructive measuring ways to measure wood MC. Many studies have been performed on the electric properties of wood, and several instruments to measure wood MC have been developed (Suzuki 2005). The sensor heads of these instruments contact with the wood surface or are inserted into the wood; thus, obtaining information about the deep portion of wood is difficult with such instruments. Consequently, the true MC of wood may be higher than the low MC indicated by these instruments. Hence, relying on measurements from these instruments can lead to deformations and cracks during storage after drying. Water distribution within logs has been estimated using an electrical impedance measurement method and the Bode plots of impedance and phase angle were used to classify MC of heartwood using the layered impedance model based on parallel resistance and capacitance (Suzuki and Ikeda 2009; Suzuki et al 2011).

Near-infrared (IR) spectroscopy is a promising nondestructive way to measure the properties of

* Corresponding author

© 2017 by the Society of Wood Science and Technology

wooden materials and enables online and onsite measurements. This method has been studied to predict the MC of wooden materials (Tsuchikawa 2011). However, estimates of MC using near-IR spectroscopy are based on information from the surface layer of <3 mm of wood (Watanabe et al 2010, 2011).

The ratio of Young's modulus by bending vibration E_B to that by longitudinal vibration E_{LV} is also a promising index to simply and nondestructively estimate the water distribution within wood. E_B is affected by the outer portion of an inhomogeneous cross-section of wood rather than the inner portion, whereas E_{LV} is the mean in the cross-section. The Young's modulus of the outer portion is larger than that of the inner portion during drying. Hence, the E_B/E_{LV} ratio increases when the outer portion of wood is sufficiently dried and the inner portion is insufficiently dried as shown by the following studies.

Tonosaki and Saito (2000) and Tonosaki et al (2001) dried 110 × 110 × 3040 mm sugi (Japanese cedar) boxed heart lumber using two drying schedules. The specimens dried equally from outside to inside using the first drying schedule, whereas the MC of the inside of the specimen was higher than that of the outside of the specimen using the second drying schedule, although the outside of the specimen for both schedules was dry. As a result, the E_B/E_{LV} ratio increased with the increase in the difference between the MC of the outer portion and that of the inner portion.

These studies investigated wood properties after drying. Hence, what process resulted in the tendencies is not shown in the previous studies (Tonosaki and Saito 2000; Tonosaki et al 2001). We think that the change in the E_B/E_{LV} ratio throughout the entire drying process should be understood to use the E_B/E_{LV} ratio as an index to simply and nondestructively estimate the water distribution within wood. In this study, the temporal change in the E_B/E_{LV} ratio was examined during the drying process.

A SIMPLE MODEL FOR AN INHOMOGENEOUS CROSS-SECTION

A temporal change in a distribution of MC in a cross section during drying can be schematically shown as Fig 1. We suppose that the Young's modulus of oven-dried wood is uniform in a cross section and the Young's modulus increases with a decrease in the MC in this study.

An inhomogeneous rectangular cross section with dimensions of $2b \times 2h$ is considered. The Young's moduli in a range of $-pb \le z \le -pb$ and $-ph \le y \le ph$ and that in the other portion are qE $(0 \le q \le 1)$ and E, respectively (Fig 2).

Young's modulus obtained by the bending vibration E_B is calculated by using the following equations:

$$E_B I = E\int_{-h}^{h} y^2 dy \int_{-b}^{-pb} dz + E\int_{-h}^{-ph} y^2 dy \int_{-pb}^{pb} dz + qE\int_{-ph}^{ph} y^2 dy \int_{-pb}^{pb} dz + E\int_{ph}^{h} y^2 dy \int_{-pb}^{pb} dz + E\int_{-h}^{h} y^2 dy \int_{pb}^{b} dz \tag{1}$$

where

$$I = \int_{-h}^{h} y^2 dA \tag{2}$$

and $A = dydz$ is the cross-section.

$$\therefore E_B = \{(q-1)p^4 + 1\}E \tag{3}$$

Young's modulus obtained using the longitudinal vibration is calculated as:

$$E_{LV} = p^2 qE + (1-p^2)E = \{(q-1)p^2 + 1\}E \tag{4}$$

$$\therefore \frac{E_B}{E_{LV}} = \frac{(q-1)p^4 + 1}{(q-1)p^2 + 1} \tag{5}$$

When E is 14 GPa (oven-dried sitka spruce, Carrington 1922; Hearmon 1948), and qE is 11 GPa (sitka spruce green wood, Carrington

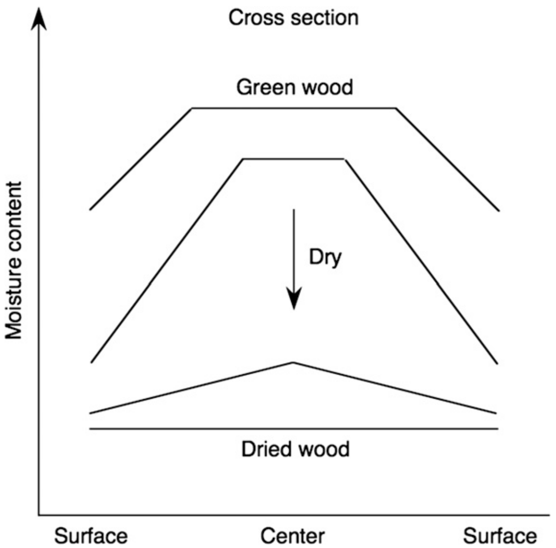

Figure 1. Schematic diagram of a temporal change in a distribution of MC in a cross section during drying.

1922; Hearmon 1948), the E_B/E_{LV} ratio is plotted against p in Fig 3.

GOENS-HEARMON REGRESSION METHOD BASED ON THE TIMOSHENKO THEORY OF BENDING

Young's and shear moduli can be obtained at the same time using only the bending vibration test without a torsional vibration test by the following the Goens-Hearmon regression method based on the Timoshenko theory of bending

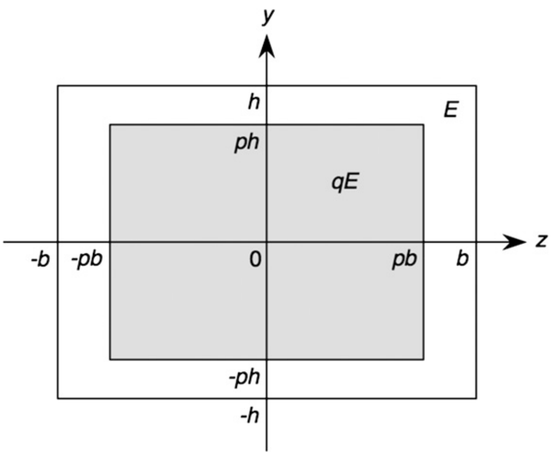

Figure 2. Simple model for an inhomogeneous cross section.

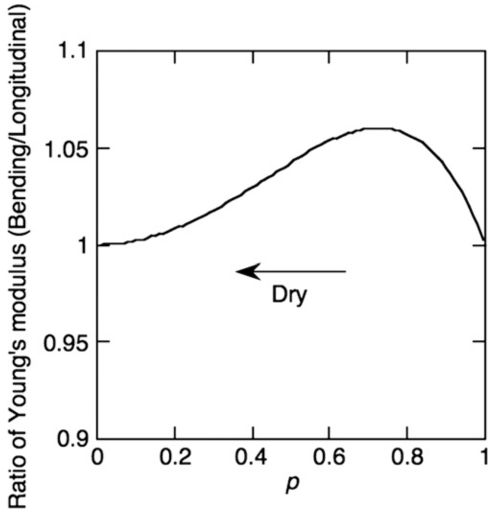

Figure 3. Change in the ratio of Young's modulus during drying (simple cross-sectional model).

(TGH method) (Kubojima et al 1996; Kubojima and Tonosaki 2013).

The apparent deflection in the bending vibration consists of shear as well as bending deflection and rotary inertia. Timoshenko (1921) added the terms of shear deflection and rotary inertia to the Euler-Bernoulli's elementary theory of bending and developed the following differential equation of bending:

$$\frac{Ei^2}{\rho}\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} - i^2\left(1 + \frac{sE}{G}\right)\frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{\rho s i^2}{G}\frac{\partial^4 y}{\partial t^4} = 0 \tag{6}$$

where E denotes Young's modulus, G is the shear modulus, i is the radius of gyration of a cross-section, ρ is the density, x is the distance along the beam, y is the lateral deflection, and t is the time.

When Eq (6) is solved under the free-free condition, the resonance frequency corresponding to the n-th mode f_n can be written as follows:

$$f_n = \frac{ik_n^2}{2\pi l^2}\sqrt{\frac{E}{\rho}} = \frac{im_n^2}{2\pi l^2}\sqrt{\frac{E_{an}}{\rho}} \tag{7}$$

where l denotes beam length and E_{an} is Young's modulus from Euler-Bernoulli's elementary theory of bending using the resonance frequency of the n-th mode and the constants m_n for the free-free bending vibration are shown by the following equation:

$$m_1 = 4.730,\ m_2 = 7.853,$$
$$m_n = \frac{1}{2}(2n+1)\,\pi(n > 2) \tag{8}$$

The value of k_n in Eq (7) is obtained by transcendental equations represented as follows:

$$\frac{\tan\dfrac{k_n}{2}\sqrt{\sqrt{B_t^2k_n^4+1}+A_tk_n^2}}{\tanh\dfrac{k_n}{2}\sqrt{\sqrt{B_t^2k_n^4+1}-A_tk_n^2}} = -\frac{\sqrt{\sqrt{B_t^2k_n^4+1}+A_tk_n^2}}{\sqrt{\sqrt{B_t^2k_n^4+1}-A_tk_n^2}}\frac{\sqrt{B_t^2k_n^4+1}-B_tk_n^2}{\sqrt{B_t^2k_n^4+1}+B_tk_n^2}\,(\text{for symmetric modes}) \tag{9}$$

and

$$\frac{\cot\dfrac{k_n}{2}\sqrt{\sqrt{B_t^2k_n^4+1}+A_tk_n^2}}{\coth\dfrac{k_n}{2}\sqrt{\sqrt{B_t^2k_n^4+1}-A_tk_n^2}} = +\frac{\sqrt{\sqrt{B_t^2k_n^4+1}+A_tk_n^2}}{\sqrt{\sqrt{B_t^2k_n^4+1}-A_tk_n^2}}\frac{\sqrt{B_t^2k_n^4+1}-B_tk_n^2}{\sqrt{B_t^2k_n^4+1}+B_tk_n^2}\,(\text{for asymmetric modes}), \tag{10}$$

where

$$A_t = \frac{i^2}{2l^2}\left(1+\frac{sE}{G}\right) \tag{11}$$

and

$$B_t = \frac{i^2}{2l^2}\left(-1+\frac{sE}{G}\right) \tag{12}$$

Goens (1931) approximated Eqs 9 and 10 by using a Taylor series into the following formula

$$\frac{m_n^4}{k_n^4} = T = 1+\frac{i^2}{l^2}\left\{m_n^2F^2(m_n)+6m_nF(m_n)\right\}+\frac{i^2}{l^2}\frac{sE}{G}\left\{m_n^2F^2(m_n)-2m_nF(m_n)\right\}-\frac{4\pi^2\rho si^2f_n^2}{G} = \frac{E}{E_{an}} \tag{13}$$

where

$$F(m_1) = 0.9825,\ F(m_2) = 1.0008,\ F(m_n) = 1(n > 2) \tag{14}$$

Hearmon (1958) calculated E and G using the following procedure after separating Eq (13) as follows:

$$Y = E_a\left[1+\frac{i^2}{l^2}\left\{m_n^2F^2(m_n)+6m_nF(m_n)\right\}-\frac{4\pi^2\rho si^2f_n^2}{G}\right] \tag{15}$$

$$X = E_a\frac{i^2}{l^2}\left\{m_n^2F^2(m_n)-2m_nF(m_n)\right\} \tag{16}$$

$$\alpha = \frac{sE}{G} \tag{17}$$

$$\beta = E \tag{18}$$

and then from Eqs 15 to 18:

$$Y = \beta - \alpha X \tag{19}$$

Therefore, the linear regression between X and Y provides the E and G values. This is the Goens-Hearmon regression method based on the Timoshenko theory of bending (TGH method). The value of s is theoretically 1.2 and experimentally 1.18 (Nakao et al 1984).

MATERIALS AND METHODS

Specimens

Sitka spruce (*Picea sitchensis* Carr.) and Japanese cedar (*Cryptomeria japonica* D. Don) were used as specimens. Small clear green wood specimens were prepared with dimensions of 300 mm (longitudinal direction, L) × 30 mm (radial direction, R) × 10 mm (tangential direction, T). Two small clear specimens were made for spruce and cedar, respectively. Large specimens of green wood were prepared with dimensions of 2000 mm (L) × 120 mm (R) × 80 mm (T) (LR specimen) and those with dimensions of 2000 mm (L) × 120 mm (T) × 80 mm (R) (LT specimen). Three LR specimens and three LT specimens were made for cedar. All specimens were left inside a controlled environment of 20°C and 65% relative humidity (RH), and the vibration tests were performed under the same conditions during drying.

Bending Vibration Test

To obtain the Young's and shear moduli by bending, free-free bending vibration tests were conducted according to the following procedure. The test beam was suspended by two threads at the free-free vibration nodal positions corresponding to its resonance mode and excited in the direction of thickness (flatwise) at one end by using a wooden hammer, whereas beam motion was detected by a microphone at the other end. The wooden hammer was used to tap the LR plane of the small clear specimen, the LR plane of the LR specimen, and the LT plane of the LT specimen. The resonance frequencies of the first to fifth modes were measured for each small clear specimen, whereas those of the first to ninth modes were measured for each LR and LT specimen, respectively. The signal was processed through a fast Fourier transform (FFT) digital signal analyzer to yield high-resolution resonance frequencies (Kubojima et al 1996; Kubojima and Tonosaki 2013) as shown in Fig 4.

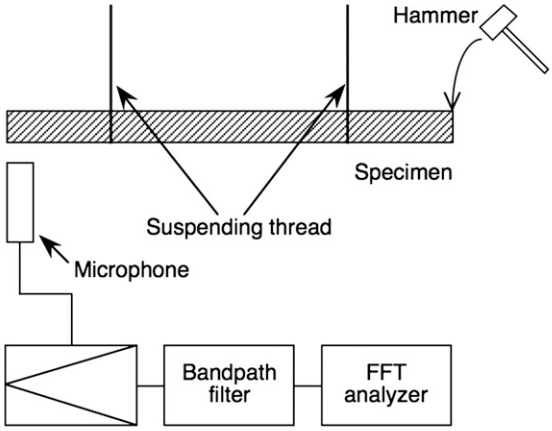

Figure 4. Schematic diagram of the experimental setup for the bending vibration test (FFT, fast Fourier transform).

Longitudinal Vibration Test

To obtain the Young's modulus, free-free longitudinal vibration tests were conducted by the following procedure. The test bar was placed on a small support at the position of $x = l/2$. A vibration was produced in the longitudinal direction at one end by a hammer, whereas

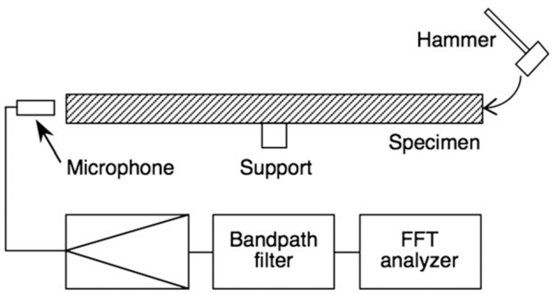

Figure 5. Schematic diagram of the experimental setup for the longitudinal vibration test (FFT, fast Fourier transform).

motion of the first mode of the bar was detected by a microphone at the other end. The signal was processed through an FFT digital signal analyzer to yield high-resolution resonance frequencies as shown in Fig 5.

RESULTS AND DISCUSSION

The TGH method uses that the Young's modulus by the Euler-Bernoulli's elementary theory E_{an} (Eq 7) decreases with an increase in the resonance mode number n due to the contribution of the shear deflection and rotary inertia to the bending deflection. When an inhomogeneity exists in a specimen, such tendency does not appear and appropriate values are not calculated from the TGH method (Kubojima et al 2005). Water is distributed in the L direction during drying, leading to an inhomogeneity of wood properties. Consequently, it is afraid that poor results are obtained from the bending vibration. Thus, whether the results from the TGH method in this study were accurate will be discussed.

The correlation coefficient of the Goens-Hearmon regression r is an index that indicates whether the TGH method is accurate. The r value is theoretically -1. As the means (standard deviations) of the absolute values of r were 0.996 (0.0046) for the small clear specimens and 0.993 (0.0047) for the LR and LT specimens, respectively, the TGH method was accurate for both sizes of specimens. This means that the water distribution in the L direction was so small that the TGH method results were unaffected.

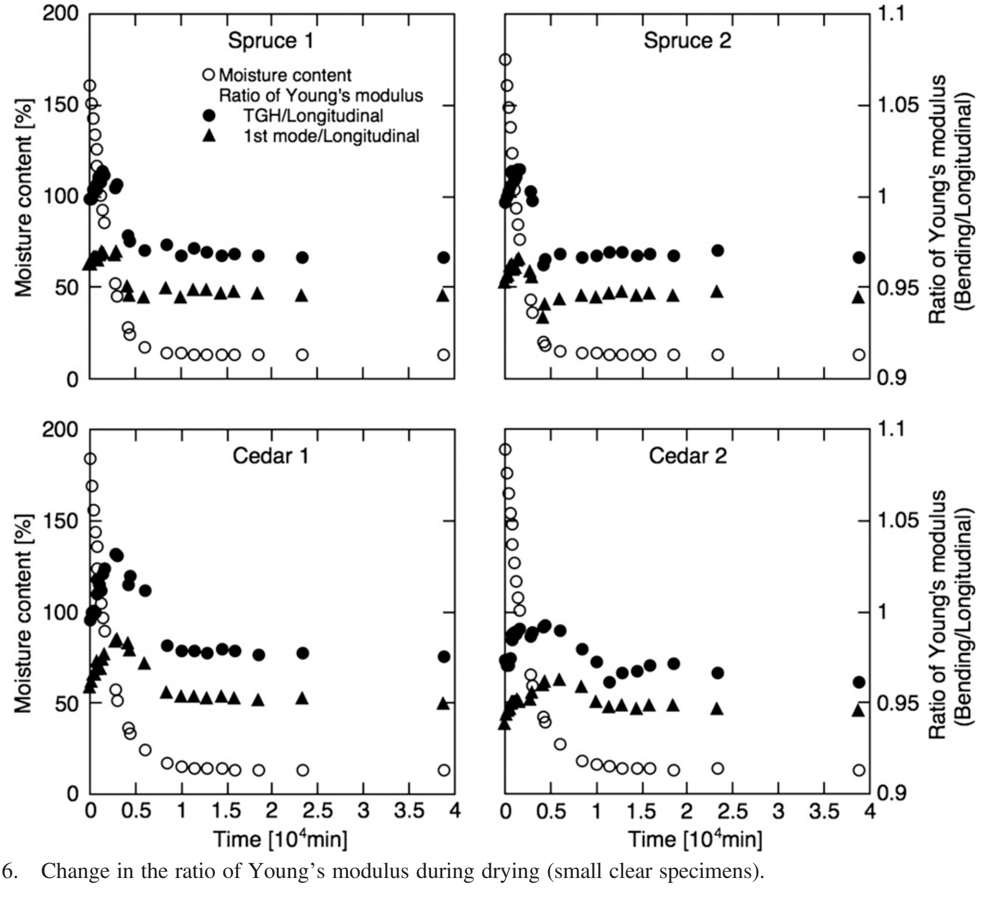

Figure 6. Change in the ratio of Young's modulus during drying (small clear specimens).

Young's modulus from the TGH method E_{TGH} is used as the Young's modulus by bending vibration E_B.

The E_{TGH}/E_{LV} ratio increased and then decreased during drying and approached 1 when the MC became constant (Figs 6 and 7). This tendency was similar to the result of the simple cross-sectional model shown in Fig 3. When a cross-section is uniform, the E_{TGH}/E_{LV} ratio should be 1. However, the E_{TGH}/E_{LV} ratio was not 1 in the

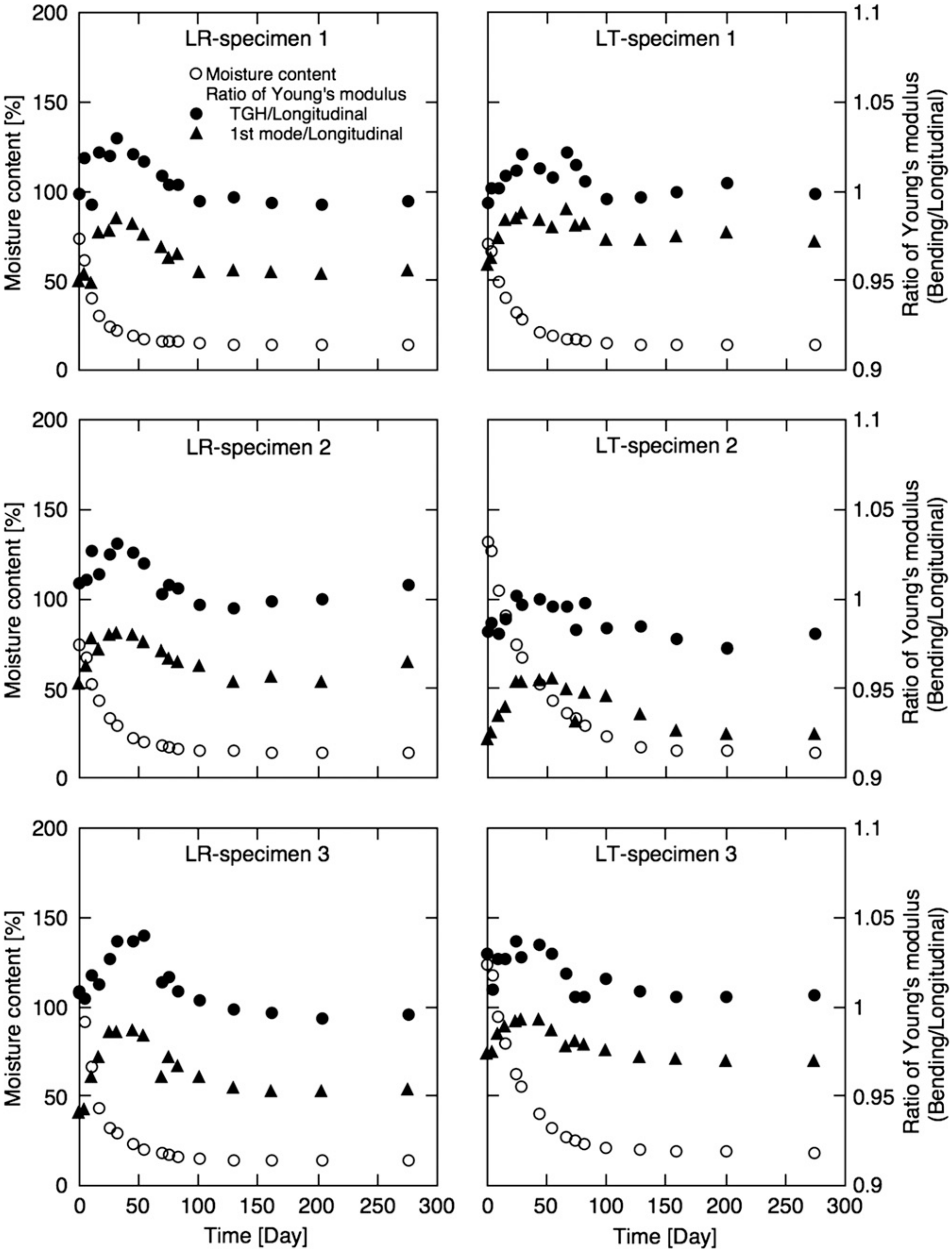

Figure 7. Change in the ratio of Young's modulus during drying (LR and LT specimens). L, longitudinal direction; R, radial direction; T, tangential direction.

first and last plots. We consider that the E_{TGH}/E_{LV} ratio was not 1 because Young's modulus varied in the cross-section under the oven-dried condition.

From Figs 6 and 7, the increases in the E_{TGH}/E_{LV} ratio before reaching the maximum show that the specimens are unsufficiently dried. As the E_{TGH}/E_{LV} ratio became constant when MC was constant, it is safe to say that the specimens are sufficiently dried when the E_{TGH}/E_{LV} ratio is constant. When the E_{TGH}/E_{LV} ratio is decreasing after reaching the maximum, whether a specimen is sufficiently dried was not clear.

The E_{TGH}/E_{LV} ratio that indicates a specimen is sufficiently dried (eg the MC $<$ FSP) could not be determined in this study. According to the previous study (Tonosaki and Saito 2000; Tonosaki et al 2001), E_B/E_{LV} ratio is large when the difference of MC outside and inside specimen is large. More studies using specimens with various sectional area and initial MC are needed to determine this value.

Although measuring resonance frequencies from several modes in a short time is difficult in a factory, measuring the resonance frequency of only the first resonance mode is possible, as used for grading machines. Hence, whether the E_{a1}/E_{LV} ratio (E_{a1}: Young's modulus obtained by Euler-Bernoulli's elementary theory using the first resonance mode, Eq 7) can be used instead of the E_{TGH}/E_{LV} ratio will be discussed.

Although E_{a1} is smaller than E_{TGH} because of the shear deflection and rotary inertia as mentioned earlier, E_{a1} should be as close as possible to E_{TGH} for the purpose of using E_{a1} instead of E_{TGH}. The contribution of shear deflection and rotary inertia to bending deflection is experimentally small when the E_{TGH}/E_{a1} ratio is $<$1.2 (Kubojima et al 1996).

The means (standard deviations) of the E_{TGH}/E_{a1} ratio were 1.03 (0.010) for the small clear specimen and 1.04 (0.010) for the LR and LT specimens, which were $<$1.2. Hence, the contribution of shear deflection to bending was small in this study. The contribution existed; however, the E_{a1}/E_{LV} ratio was smaller than the E_{TGH}/E_{LV} ratio. As the temporal changes in the E_{a1}/E_{LV} ratio were similar to those in the E_{TGH}/E_{LV} ratio as shown Figs 6 and 7, the E_{a1}/E_{LV} ratio could be used instead of the E_{TGH}/E_{LV} ratio.

The E_{TGH}/E_{LV} ratio approached 1.3 in previous studies (Tonosaki and Saito 2000; Tonosaki et al 2001), but was about 5% different in the present study. The specimens in the previous studies were boxed heart lumber. Hence, we consider that the Young's modulus in the previous studies varied in a cross-section under the oven-dried condition rather than that in this study.

CONCLUSIONS

Temporal changes in the ratio of Young's modulus by the bending vibration test to that by the longitudinal vibration test during leaving the specimens inside a controlled environment of 20°C and 65% RH were investigated to estimate water distribution within wood lumber. The results obtained were as follows.

1) The E_{TGH}/E_{LV} ratio increased and then decreased during drying due to the difference between the Young's modulus of the outer portion of wood and that of the inner portion.
2) The temporal change in the E_{TGH}/E_{LV} ratio was similar to the estimate using the simple cross-sectional model developed in this study.
3) The bending Young's modulus calculated using the Euler-Bernoulli's elementary theory could be used instead of that using the TGH method.
4) The changes in Young's modulus were smaller than those of previous studies.

ACKNOWLEDGMENTS

This study was financially supported by the Research grant #201402 of the Forestry and Forest Products Research Institute and grants from the Project of the Bio-oriented Technology Research Advancement Institution, NARO (the special scheme project on advanced research and development for next-generation technology).

REFERENCES

Carrington H (1922) The elastic constants of spruce as influenced by moisture. Aeronaut J 26(144):462-471.

Goens E (1931) Über die Bestimmung des Elastizitätsmodulus von Stäben mit Hilfe von Biegungsschwingungen. Annal Phys 403(6):649-678.

Hearmon RFS (1948) The elasticity of wood and plywood. His Majesty's Stationary Office, London, United Kingdom.

Hearmon RFS (1958) The influence of shear and rotatory inertia on the free flexural vibration of wooden beams. Br J Appl Phys 9(10):381-388.

Kubojima Y, Tonosaki M (2013) Effect of specimen width on the shear modulus of wood obtained by flexural vibration tests. Wood Fiber Sci 45(2):170-177.

Kubojima Y, Tonosaki M, Yoshihara H (2005) Effect of additional mass on Young's modulus of a wooden beam. J Test Eval 33(4):278-282.

Kubojima Y, Yoshihara H, Ohta M, Okano T (1996) Examination of the method of measuring the shear modulus of wood based on the Timoshenko theory of bending. Mokuzai Gakkaishi 42(12):1170-1176.

Nakao T, Okano T, Asano I (1984) Measurement of anisotropic shear modulus by the torsional-vibration method for free-free wooden beams. Mokuzai Gakkaishi 30(11): 877-885.

Suzuki Y (2005) Evaluation of moisture content of wood by electrical measurement. Wood Industry 60(10): 483-488.

Suzuki Y, Ikeda K (2009) Estimation of water distribution in sugi (Japanese cedar) log using impedance models. Proc 8th International Conference on Electromagnetic Wave Interaction with Water and Moist Substances: 289-294.

Suzuki Y, Ikeda K, Woodhead I (2011) Evaluation of moisture content gradient in a log using impedance models. Proc 9th International Conference on Electromagnetic Wave Interaction with Water and Moist Substances: 42-46.

Timoshenko SP (1921) On the correction for shear of the differential equation for transverse vibrations of prismatic bars. Phil Mag 6 Ser 41(245):744-746.

Tonosaki M, Saito S (2000) Evaluation of non-homogeneity in wood by longitudinal and flexural vibration tests I. Trial on estimation of inner moisture content of dry-processed sugi boxed heart square sawn timber. Mokuzai Gakkaishi 46(1):20-24.

Tonosaki M, Saito S, Hiramatsu Y (2001) Evaluation of non-homogeneity in wood by longitudinal and flexural vibration tests II. Distribution of vibrational properties and FEM simulation of sugi boxed heart square sawn timber. Mokuzai Gakkaishi 47(2):92-96.

Tsuchikawa S (2011) Nondestructive measurement of wooden materials by near infrared spectroscopy: Present conditions and prospects. Wood Industry 66(10):430-435.

Watanabe K, Hart F, Mansfield SD, Avramidis S (2010) Detection of wet-pockets on the surface of *Tsuga heterophylla* (Raf.) Sarg. by near infrared (NIR) spectroscopy. Holzforschung 64(1):55-60.

Watanabe K, Mansfield SD, Avramidis S (2011) Application of near-infrared spectroscopy for moisture-based sorting of green hem-fir timber. J Sood Sci 57(4): 288-294.